

CLAIMS:

In The Claims:

1. A calculator implemented resolution method for an optimization problem, the problem being modeled with constrained discrete variables, the variables having a referenced set of possible states, the calculator implemented method comprising:
  - (a) performing survey propagation in which a graph of the problem and warnings are determined, a warning giving information on whether the various assignments of the related variable are compatible with the constraints involving this variable, and in that messages are exchanged within the graph in order to determine, for each constraint and each variable involved in this constraint, a message sent from the constraint to the variable containing a set of probabilities for the various patterns of warnings that the variable can receive,
  - (b) performing survey induced decimation in which, using the messages, a degree of polarization for a variable being computed by first establishing a list of numbers, each one giving how favorable it is to assign the variable to one state of its set of possible states, each of the numbers being computed from the probability of warnings determined in the messages, the list being used to compute a degree of polarization by determining to what degree the most favorable assignment in the list is better than all other possible assignments, and at least one of the variables being assigned one state of its set of possible states according to its degree of polarization as to produce a simplified problem,
  - (c) repeating steps (a) and (b) with the simplified problem till all variables are either assigned or are unpolarized.

2. The method according to claim 1, the survey propagation step further comprising:

determining from the modelisation of the problem an interaction graph with edges linking nodes, the nodes being the variables,  $S_1, S_2, S_3\dots$ , and the constraints,  $a, b, c, d\dots$ , a variable node being connected by an edge to its related constraint(s), the interaction graph being bipartite, a variable being connected only to constraints, a constraint being connected only to variables, and for each constraint which is linked to a variable, updating the graph by:

determining over the graph, first a list of elementary messages called cavity-bias sent from each constraint to its related variable, the cavity bias being a message having a number of binary items equal to the number of possible states of the variable to which it is sent, each binary item being either void or non-void, the void corresponding to an absence of constraint on the corresponding state of the variable and the non-void corresponding to the reverse, the cavity biases being initialized to random values,

determining over the graph, a list of second elementary messages called cavity-fields sent from each variable to its related constraints, the cavity-field being a message having a number of integer items equal to the number of possible states of the variable which sends it, each integer item value being the number of non-void received from all cavity-biases to said variable for the referenced possible state of the said variable,

determining over the graph, a list of local-fields which are sets of integer values in relation to variables, each local-field being a set having a number of integer values equal to the number of possible states of the variable and each integer value being the number of non-void received by the variable in cavity-biases for each possible state of the variable,

the cavity-bias sent from a constraint  $a$  to a variable  $S$  being computed on the constraint  $a$  from the cavity-field(s) received by

said constraint  $a$  from all the other variables to which said constraint  $a$  is linked, thus excluding  $S$ , and, for each of said cavity-fields, the least penalized subspace of possible states of the variable is determined as being a set of the references of possible states for which the number of non-void is minimal, then a truth table restricted to said sets for all said cavity-fields and for all the references of possible states for the variable  $S$  is created in relation to the constraint  $a$ , from this restricted truth table a void is assigned in the cavity-bias for the referenced possible state of the variable  $S$  if the constraint is satisfied and a non-void if the constraint is not satisfied;

determining over the graph, probability laws of each cavity-bias sent from a constraint  $a$  to a linked variable  $S$  with  $q$  possible states and called cavity-bias-surveys, a cavity-bias-survey being a set of  $2^q$  probabilities for each possible configuration of its cavity-bias,

determining over the graph, probability laws of each cavity-field sent from a variable  $S$  to a linked constraint  $a$  and called cavity-field-surveys, a cavity-field-survey being a set of probabilities for each admissible configuration of its cavity-field, an admissible configuration of cavity-field being one with at least one void,

the cavity-bias-survey sent from a constraint  $a$  to a variable  $S$  being computed on the constraint  $a$  from the cavity-field-survey(s) received by said constraint  $a$  from all the other variables to which said constraint  $a$  is linked, thus excluding  $S$ , by using a look-up table characterizing constraint  $a$ , said look-up table being a list giving, for each possible assignment of all variables participating to the constraint, whether the constraint is satisfied by the assignment or not, and computing the probability that the constraint is unsatisfied, for each state of the variable,

previous survey propagation steps updates being run successively on the constraints and variables along the graph, said updates being stopped after a predetermined number of updates if it is not possible to find a set of cavity-bias-surveys which does not change, when one round of updates on all constraints and on all variables participating to the constraints is performed within a given preassigned resolution and, then, being restarted from the beginning with cavity biases initialized to new random values,

the survey induced decimation step further comprising:

determining over the graph, a local-field-survey for each variable which is a probability law of all possible local-field by computing for each variable  $S$  from all the cavity-bias-surveys received by said variable and for each possible state of said variable the joint probability of each admissible local-field, an admissible local-field being one with at least one zero value, and with the previously determined local-field-surveys: determining the degree of polarization of each variable by computing, for each assignment of the variable, the probability of having zero value as given by the local-field-survey, and computing for each assignment of the variable, the maximum of this probability diminished by the sum of the probabilities for all other assignments

the variable with the largest degree of polarization is assigned to its preferred state, the one with the largest probability of having zero value as given by the local-field-survey, the constraints containing this assigned variable are reduced, those which are satisfied are eliminated, in order to produce the simplified problem, and

the repeating step further comprising:

repeating steps (a) and (b) with the simplified problem until all variables are assigned or are unpolarized, meaning that, for all the possible assignments of the variable, the probabilities of having

zero value as given by the local-field-survey, diminished by the sum of the probabilities of having zero value as given by the local-field-survey for all other assignments, are equal within a predetermined resolution.

3. The method according to claim 2, further comprising, in that in case the determination of the cavity-field-surveys is not possible, none of the possible configuration of the cavity-field having a void, a penalty function is used for selecting the admissible configurations and that same penalty function is used for the determination of the local-field-surveys.

4. The method according to claim 3, the penalty function being an exponential of the type  $w=\exp(-yh)$  where  $y$  is the amount of penalty and  $h$  the minimum of the values of the cavity-field for all possible states of the variable.

5. The method according to claim 4, further comprising, in case the survey propagation steps updates are stopped and restarted because it is not possible to find a set of cavity-bias-surveys which does not change, reducing the penalty amount  $y$ .

6. The method according to claim 2, further comprising, choosing at random the constraint and the variable to update over the graph.

7. The method according to claim 2, further comprising, choosing the constraint and the variable to update over the graph sequentially within a randomized list of the constraints and variables.

8. The method according to claim 2, further comprising, choosing the constraint and the variable to update over the graph by constraints, all the steps being done for all variables related to said constraint before choosing another constraint.

9. An apparatus for solving a constraint satisfaction problem, the problem being modeled with constrained discrete variables

having a referenced set of possible states, the apparatus comprising:

- (a) a survey propagation device in which a graph of the problem and warnings are determined, a degree of polarization for a variable being computed by first establishing a list of numbers, each one giving how favorable it is to assign the variable to one state of its set of possible states, each of these numbers being computed from the probability of warnings determined in the previous messages, the list being used to compute the degree of polarization by determining to what degree the most favorable assignment in the list is better than all other possible assignments, and in that messages are exchanged within the graph in order to determine, for each constraint and each variable involved in this constraint, a message sent from the constraint to the variable containing a set of probabilities for the various patterns of warnings that the variable can receive,
- (b) a survey induced decimation device in which, using the messages, a degree of polarization of each variable is determined, a degree of polarization for a variable being computed by first establishing a list of numbers, each one giving how favorable it is to assign the variable to one state of its set of possible states, each of these numbers being computed from the probability of warnings determined in the previous messages, the list being used to compute the degree of polarization by determining to what degree the most favorable assignment in the list is better than all other possible assignments, and at least one variable is assigned one state of its set of possible states according to its degree of polarization as to simplify the problem,
- (c) the survey propagation device and the survey induced decimation device being operated with the simplified problem until all variables are either assigned or are unpolarized.

10. Device according to claim 9, the survey propagation device, further comprising:

means for determining from the modelisation of the problem an interaction graph with edges linking nodes, the nodes being the variables,  $S_1, S_2, S_3\dots$ , and the constraints,  $a, b, c, d\dots$ , a variable node being connected by an edge to its related constraint(s), the interaction graph being bipartite, a variable being connected only to constraints, a constraint being connected only to variables, and for each constraint which is linked to a variable, updating the graph,

means for determining over the graph, first a list of elementary messages called cavity-bias sent from each constraint to its related variable, the cavity bias being a message having a number of binary items equal to the number of possible states of the variable to which it is sent, each binary item being either void or non-void, the void corresponding to an absence of constraint on the corresponding state of the variable and the non-void corresponding to the reverse, the cavity biases being initialized to random values,

means for determining over the graph, a list of second elementary messages called cavity-fields sent from each variable to its related constraints, the cavity-field being a message having a number of integer items equal to the number of possible states of the variable which sends it, each integer item value being the number of non-void received from all cavity-biases to said variable for the referenced possible state of the said variable,

means for determining over the graph, a list of local-fields which are sets of integer values in relation to variables, each local-field being a set having a number of integer values equal to the number of possible states of the variable and each integer value being the number of non-void received by the variable in cavity-biases for each possible state of the variable,

the cavity-bias sent from a constraint a to a variable S being computed on the constraint a from the cavity-field(s) received by said constraint a from all the other variables to which said constraint a is linked, thus excluding S, and, for each of said cavity-fields, the least penalized subspace of possible states of the variable is determined as being a set of the references of possible states for which the number of non-void is minimal, then a truth table restricted to said sets for all said cavity-fields and for all the references of possible states for the variable S is created in relation to the constraint a, from this restricted truth table a void is assigned in the cavity-bias for the referenced possible state of the variable S if the constraint is satisfied and a non-void if the constraint is not satisfied,

means for determining over the graph, probability laws of each cavity-bias sent from a constraint a to a linked variable S with q possible states and called cavity-bias-surveys, a cavity-bias-survey being a set of  $2^q$  probabilities for each possible configuration of its cavity-bias,

means for determining over the graph, probability laws of each cavity-field sent from a variable S to a linked constraint a and called cavity-field-surveys, a cavity-field-survey being a set of probabilities for each admissible configuration of its cavity-field, an admissible configuration of cavity-field being one with at least one void,

the cavity-bias-survey sent from a constraint a to a variable S being computed on the constraint a from the cavity-field-survey(s) received by said constraint a from all the other variables to which said constraint a is linked, thus excluding S, by using a look-up table characterizing constraint a, said look-up table being a list giving, for each possible assignment of all variables participating to the constraint, whether the constraint is satisfied

by the assignment or not, and computing the probability that the constraint is unsatisfied, for each state of the variable,

the previous survey propagation steps updates being run successively on the constraints and variables along the graph, said updates being stopped after a predetermined number of updates if it is not possible to find a set of cavity-bias-surveys which does not change, when one round of updates on all constraints and on all variables participating to the constraints is performed, within a given preassigned resolution and, then, being restarted from the beginning with cavity biases initialized to new random values, and

the survey induced decimation device further comprising:

means for determining over the graph, a local-field-survey for each variable which is a probability law of all possible local-field by computing for each variable  $S$  from all the cavity-bias-surveys received by said variable and for each possible state of said variable the joint probability of each admissible local-field, an admissible local-field being one with at least one zero value, and, with the previously determined local-field-surveys:

means for determining the degree of polarization of each variable by computing, for each assignment of the variable, the probability of having zero value as given by the local-field-survey, and computing for each assignment of the variable, the maximum of this probability diminished by the sum of the probabilities for all other assignments, the variable with the largest degree of polarization being assigned to its preferred state, the one with the largest probability of having zero value as given by the local-field-survey, the constraints containing this assigned variable being reduced, those which are satisfied are eliminated, in order to produce the simplified problem, and

means for operating the survey propagation device and survey induced decimation device with the simplified problem until

all variables are assigned or are unpolarized, meaning that, for all the possible assignments of the variable, the probabilities of having zero value as given by the local-field-survey, diminished by the sum of the probabilities of having zero value as given by the local-field-survey for all other assignments, are equal within a predetermined resolution.

11. The apparatus according to claim 10, further comprising, means in case the determination of the cavity-field-surveys is not possible, none of the possible configuration of the cavity-field having a void, for performing a penalty function for selecting the admissible configurations and that same penalty function is used for the determination of the local-field-surveys.

12. The apparatus according to claim 11, the penalty function being an exponential of the type  $w=\exp(-yh)$  where  $y$  is the amount of penalty and  $h$  the minimum of the values of the cavity-field for all possible states of the variable.

13. The apparatus according to claim 12, further comprising, means in case the survey propagation device updates are stopped and restarted because it is not possible, for finding a set of cavity-bias-surveys which does not change, the penalty amount  $y$  is reduced.

14. The apparatus according to claim 10, further comprising, means to choose at random the constraint and the variable to update over the graph.

15. The apparatus according to claim 10, further comprising means to choose sequentially within a randomized list of the constraints and variables, the constraint and the variable to update over the graph.

16. The apparatus according to claim 10, further comprising means to choose by constraints the constraint and the variable to update over the graph, all the steps being done for all variables related to said constraint before choosing another constraint.

17. A calculator readable media which tangibly embodies calculator instructions intended to operate the calculator according to the any one of claims 1 to 8.